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On Semi-Essential Subsemimodules in Multiplication Semimodules

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ABSTRACT

The Semi essential subsemimodule was defined in [4]. In this paper we generalize some results of semi essential submodule to semi essential subsemimodules in multiplication semimodule.

Indexing terms/Keywords

Semiring; semimodule; essential subsemimodule; semi-essential subsemimodule.

SUBJECT CLASSIFICATION

2010 AMS Classification 2010: 16Y60.

1. INTRODUCTION

In [4]-[7] the author has investigated and studied different classes of essential ideals and essential subsemimodules. The notion of semi-essential subsemimodule was introduced in [6] by Kishor Pawar and Pritam Gujarathi. In this paper we generalize the properties of semi-essential module over multiplication semimodule [1] and also we give conditions when an R-subsemimodule of multiplication subsemimodule becomes semi-essential subsemimodule.

2. PRELIMINARIES

Definition 2.1. [3] A semiring is a set R together with two binary operations called addition $(+)$ and multiplication $(\cdot)$ such that $(R,+)$ is a commutative monoid with identity element 0_R; $(R,\ \cdot)$ is a monoid with identity element 1; multiplication distributes over addition from either side and 0 is multiplicative absorbing, that is, $a\cdot 0 = 0\cdot a = 0$ for each $a\in R$. A semiring R is said to have a unity if there exists $1_R\in R$ such that $1_R\cdot a = a\cdot 1_R = a$ for each $a\in R$.

For e.g.: The set $\mathbb{N}$ of non-negative integers with the usual operations of addition and multiplication of integers is a semiring with $1_{\mathbb{N}}$.

Definition 2.2. [3] Let R be a semiring. A left R-semimodule is a commutative monoid $(M,+)$ with additive identity 0_M for which we have a function $R\times M\to M$ defined by $(r,m)\hookrightarrow r$ ·m and called scalar multiplication which satisfies the following conditions for all r and r' of R and all elements m and m' of M,

1. $(r\cdot r')m = r(r'\cdot m)$
2. $r\cdot(m+m') = r\cdot m + r\cdot m'$
3. $(r+r')\cdot m = r\cdot m + r'\cdot m$
4. $1_R\cdot m = m$ (If exists)
5. $r\cdot 0_M = 0_M = 0_R\cdot m$.

Convention: In this paper all semirings considered will be assumed to be commutative semirings with unity.

Definition 2.3 [2]: Let R be a semiring and M be an R-semimodule. A subsemimodule N of M is called prime if

i) N is proper subsemimodule of M and

ii) If for any $m \epsilon M, r \in R,\ mr \in N \ \Rightarrow m \in N$ or $r \in A_N(M) = \{a \in R \mid aM \subseteq N\}$.

Definition 2.4 [3]: A nonzero R-subsemimodule N of M is called semi-essential if $N \cap P \neq 0$ for each nonzero prime R-subsemimodule P of M.

3. SEMI-ESSENTIAL SUBSEMIMODULES IN MULTIPLICATION SEMIMODULES

In this section, we give a condition under which an R-subsemimodule N of a faithful multiplication R-semimodule M becomes semi essential.

Definition 3.1.[3] An R-semimodule M is called a multiplication semimodule where N is a subsemimodule of M, then there exists an ideal I of R such that

$N = IM.\ I = (N : M) = \{r \in R / rm \subseteq N\}$

Proposition 3.2[6]: A nonzero R-subsemimodule N of M is semi-essential if and only if for each nonzero prime R-subsemimodule P of M there exists $x \in P$ and there exists $r \in R$ such that $0 \neq rx \in N$.

Proposition 3.3[6]: Let M be an R-subsemimodules and let N_1 and N_2 be R-subsemimodules of M such that N_1 is an R-subsemimodules of N_2. If N_1 is a semi-essential R-subsemimodule of M, then N_2 is a semi-essential R-subsemimodule of M.

Corollary 3.4 [6]: Let N_1 and N_2 are R-subsemimodules of M. If $N_1 \cap N_2$ is a semi-essential R-subsemimodule of M, then N_1 and N_2 are semi-essential.

Proposition 3.5[6]: Let N_1 and N_2 are R-subsemimodules of M such that N_1 is essential and N_2 is semi-essential. Then $N_1 \cap N_2$ is a semi-essential R-subsemimodule of M.

Lemma 3.6 [6]: Let N be an R-subsemimodule of M and let P be a prime subsemimodule of M. If $(N \cap P : x) = ann(M)$, for each $x \in M$ and $x \notin N \cap P$, then $N \cap P$ is a prime R-subsemimodule of M.

Theorem 3.7 [8] If M is finitely generated multiplication semimodule over a semiring R, P is a strong k-ideal of R containing $ann(M)$, then PM is a prime subsemimodule of M.

Theorem 3.8. Let M be a faithful multiplication R-semimodule and N is an R-subsemimodule of M such that $N = IM$ for some ideal I of R. If N is semi essential if and only if I is semi essential with $I \bigcap P = 0$, where P is strong prime k-ideal of R containing $ann(M)$.

Proof Since M is faithful multiplication R-semimodule then $(I \bigcap P)M = 0$ Implies $IM \bigcap PM = 0$.

PM is prime R-subsemimodule of M and $N = IM$ is semiessential R subsemimodule of M therefore $PM = 0$. Implies $P = 0$. Hence I is semiessential ideal of R.

Conversely, Let $N \bigcap P = 0$, where P is non zero prime R-subsemimodule of M. Since M is multiplication semimodule there exists a strong prime k-ideal P' of R such that $P = P'M$. Hence $N \bigcap P = IM \cap P'M = (I \cap P')M = 0$. But M is faithful implies $I \cap P' = 0$. Since I is semi essential ideal of R, then $P' = 0$. Therefore $P = 0$ implies N is semiessential R subsemimodule of M.

Theorem 3.9: Let M be a faithful multiplication R-semimodule Then N is a semiessential R-subsemimodule of M if and only if $(N : x)$ is a semi-essential ideal of R for each $x \in M$

Proof: Suppose that N is semi essential. By above Theorem 2.7 M is faithful multiplication of R semimodule then $(N : M)$ is semi essential k-ideal of R. But $(N : M) \subseteq (N : x)$. Therefore for each $x \in M, N = (N : M)M \subseteq (N : x)M$. Implies $(N : x)M$ is semi essential R-subsemimodule of M

And consequently $(N : x)$ is a semiessential k-ideal of R.

Conversely assume that $(N : x)$ is semi essential k-ideal of R for each $x \in M$. Let P be a nonzero prime R-submodule of M and let $0 \neq y \in P$. Thus $(N : y)$ is semi essential. Since M is multiplication then $P = P'M$, where P is a strong k-prime ideal of R. Hence $(\mathrm{N} : \mathrm{y}) \cap \mathrm{P}' \neq 0$. By assumption M is faithful, So $(N : x)M \cap P'M \neq 0$ Thus $N \cap P \neq 0$.

Proposition 3.10: Let M be a faithful multiplication R-semimodule and let N be nonzero prime R-subsemimodule of M. If N is not minimal prime, then N is semiessential.

Proof: Since M is multiplication and N is prime, then there exists a strong prime k-ideal P' of R such that $ann(M) \subseteq P'$ such that $N = P'M$.Let P be nonzero prime R-subsemimodule of M such that $N \cap P = 0$. Since N is not minimal prime there exists a minimal prime R-subsemimodule L of M such that $L \subset N$.Thus there exists a strong minimal prime ideal P'' of R such that $ann(M) \subseteq P''$ and $L = P''M \neq M$. Rest of the proof is same as in ring.

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